

# Towards an unified theoretical model of ship dynamics

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## Introduction

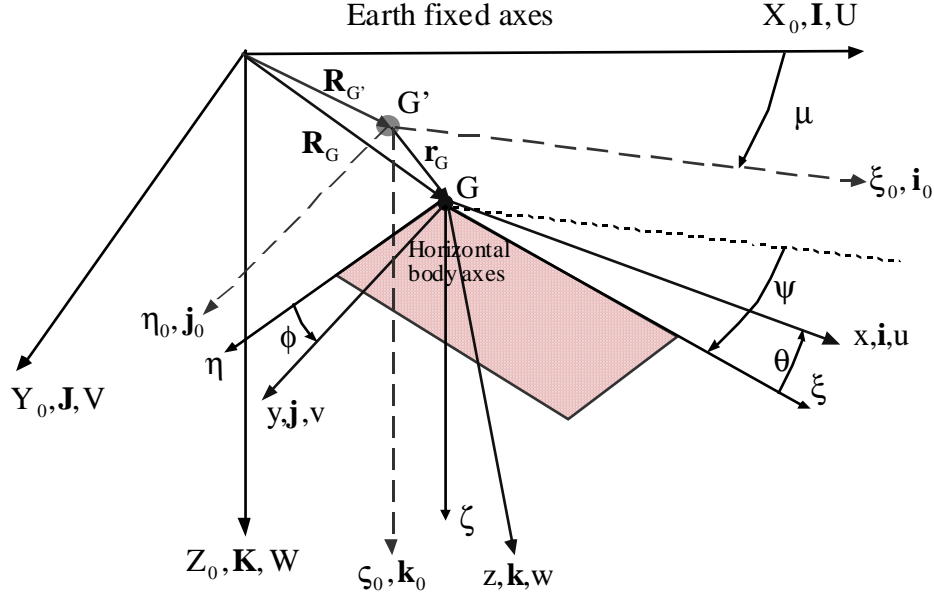
Linear models of ship dynamics in waves are well established. In most cases they result in a sufficiently accurate prediction of loads and ship motions. Perhaps the biggest benefit of using the linear models is that prediction of exceeding certain level of load or response can be easily derived. Analysis is conveniently conducted in the frequency domain. The biggest shortcoming of the linearity assumption is that it precludes prediction of certain classes of ship responses. The linear models can not predict the loss of ship stability in waves, parametric resonance of roll and asymmetry of sagging and hogging. Ship steering and manoeuvring motion are disregarded.

Simulation of ship manoeuvring is usually conducted for the still water condition. Time-domain simulation of ship motion is restricted to in-plane motion comprising of surge, yaw and sway motion components. If waves are encountered for, their effect is taken into account as a steady state one.

The method which evaluates in time-domain ship rigid body motions in waves and manoeuvring is presented briefly. The so called two-stage approach (Matusiak, 2000, Matusiak, 2001) is used when evaluating non-linear responses. The method preserves best features of the linear seakeeping theory and takes into account most important non-linearities. As a result the loss of ship stability in waves and parametric resonance of roll are numerically predicted. Ship manoeuvring in regular waves can be simulated, too.

## Six-degrees-of-freedom general model of ship motion

Ship is regarded as a rigid body possessing in general six degrees of freedom. Four co-ordinate systems are used for describing general ship motion. These are presented in Figure 1.



**Fig. 1** Co-ordinate systems used in ship dynamics.

Inertial co-ordinate system fixed to Earth is denoted by  $X_0Y_0Z_0$ .  $X_0$ -axis points in the wave propagation direction. The  $X_0$ - $Y_0$  plane coincides with the still water level. Ship is on course  $\mu$  with respect to waves. Course or encounter angle is a time-averaged or initial orientation of ship with respect to the direction of wave propagation. This time-averaged position defines the co-ordinate system  $\xi_0\eta_0\zeta_0$ .  $G'$  is the origin of this co-ordinate system and it is the time-averaged position of the ship's center of gravity. Axis  $\xi_0$  points in the direction of ship velocity vector  $V_s$ . The average position of ship is given by the position vector  $\mathbf{R}_{G'} = X_G\mathbf{I} + Y_G\mathbf{J}$ .

The origin of two other Cartesian co-ordinate systems is located at the instantaneous position of ship's origin (point  $G$  in Fig.1). Co-ordinate system  $xyz$  is fixed to the ship so that the  $x$ -axis points towards ship bow. This co-ordinate system is called the body-fixed co-ordinate system. The so-called horizontal body axes co-ordinate system (Hamamoto, 1993) denoted as  $\xi\eta\zeta$  moves with ship so that the  $\xi$ - $\eta$  plane stays horizontal that is it is parallel to the plane  $X_0$ - $Y_0$  and  $\zeta$ -axis stays at ship centreplane. Both the body fixed and horizontal axes co-ordinate systems move with ship with a velocity  $\mathbf{U}$ .

Instantaneous position of ship's center of gravity is given by the following displacement components: surge ( $\xi_0$  or  $x_1$ ), sway ( $\eta_0$  or  $x_2$ ) and heave ( $\zeta_0$  or  $x_3$ ). These are the motion components of the center of gravity in the moving with ship velocity  $V_s$  inertial co-ordinate system  $\xi_0\eta_0\zeta_0$ . Translational motion is defined as the motion of ship's origin  $0$  in the inertial co-ordinate system

$$\mathbf{r}_G = \xi_0\mathbf{i}_0 + \eta_0\mathbf{j}_0 + \zeta_0\mathbf{k}_0. \quad (1)$$

The velocity of the origin of ship is given as

$$\mathbf{U} = \dot{\mathbf{r}}_G = \dot{\xi}_0 \mathbf{i}_0 + \dot{\eta}_0 \mathbf{j}_0 + \dot{\zeta}_0 \mathbf{k}_0 = u\mathbf{i} + v\mathbf{j} + w\mathbf{k} . \quad (2)$$

Angular position of the ship is given by the so-called ship Euler angles denoted in Fig. 1 as  $\psi, \theta$  and  $\phi$ . The following matrix relation (Clayton&Bishop 1982; Fossen, 1994) gives the projection of the velocity expressed in body-fixed co-ordinate system on the Earth-fixed co-ordinates

$$\begin{Bmatrix} \dot{\xi}_0 \\ \dot{\eta}_0 \\ \dot{\zeta}_0 \end{Bmatrix} = \begin{bmatrix} \cos\psi\cos\theta & \cos\psi\sin\theta\sin\phi & \cos\psi\sin\theta\cos\phi \\ \sin\psi\cos\theta & \sin\psi\sin\theta\sin\phi & \sin\psi\sin\theta\cos\phi \\ -\sin\theta & \cos\theta\sin\phi & \cos\theta\cos\phi \end{bmatrix} \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} \quad (3)$$

Angular velocity  $\boldsymbol{\Omega}$  of ship can be expressed in terms of the time derivatives of roll, pitch and yaw as follows

$$\boldsymbol{\Omega} = P\mathbf{i} + Q\mathbf{j} + R\mathbf{k} . \quad (4)$$

The dependence of the derivatives of the Euler angles and angular velocity components expressed in the moving frame is as follows (Clayton&Bishop, 1982)

$$\begin{Bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{Bmatrix} = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi/\cos\theta & \cos\phi/\cos\theta \end{bmatrix} \begin{Bmatrix} P \\ Q \\ R \end{Bmatrix} . \quad (5)$$

Equations of motion are given by the set of six non-linear 2<sup>nd</sup> order ordinary differential equations (Fossen, 1994)

$$\begin{aligned} X_g - mg\sin\theta &= m(\dot{u} + Qw - Rv) \\ Y_g + mg\cos\theta\sin\phi &= m(\dot{v} + Ru - Pw) \\ Z_g + mg\cos\theta\cos\phi &= m(\dot{w} + Pv - Qu) \end{aligned}$$

$$\begin{aligned}
K_g &= I_x \ddot{X} - I_{xy} \ddot{Y} - I_{xz} \ddot{Z} + (I_z R - I_{zx} P - I_{zy} Q) Q - (I_y Q - I_{yz} R - I_{yx} P) R \\
M_g &= -I_{yx} \ddot{X} + I_y \ddot{Y} - I_{yz} \ddot{Z} + (I_x P - I_{xy} Q - I_{xz} R) R - (I_z R - I_{zx} P - I_{zy} Q) P \\
N_g &= -I_{zx} \ddot{X} - I_{zy} \ddot{Y} + I_z \ddot{Z} + (I_y Q - I_{yz} R - I_{yx} P) P - (I_x P - I_{xy} Q - I_{xz} R) Q.
\end{aligned} \quad (6)$$

In equations (6),  $X_g$ ,  $Y_g$ ,  $Z_g$ ,  $K_g$ ,  $M_g$  and  $N_g$  depict the components of global reaction force and moment vectors acting on the ship. These are given in the-body fixed co-ordinate system xyz. In general these forces are non-linear. Mass of ship and the components of the mass moment of inertia are depicted by  $m$  and  $I_{ij}$ .

Apart non-linearities of the left-hand-side of equations (6) also the body dynamics model comprises of non-linear cross-coupling terms.

### Approximate models of maneuvering and seakeeping

In the maneuvering simulation normally 1<sup>st</sup>, 2<sup>nd</sup> and 6<sup>th</sup> equation of the equations set (6) are considered only. Non-linear cross-coupling terms are preserved. The hydrodynamic forces acting on a hull are represented by the experimentally derived quantities. The actions of propeller and rudder are taken into account by relatively simple semiempirical models.

Linear seakeeping theory is based on the full linearization of equations (6). The dynamics model is based on the small oscillation assumption. Non-linear cross-coupling terms are dropped out. The action of propeller and rudder is disregarded. Hydrostatic and hydrodynamic forces are represented by the linear approximations. Relatively sophisticated analytical models of wave and radiation forces are used. The linear seakeeping approximation to the equations of motion can be expressed as follows

$$\begin{aligned}
m \ddot{X}_L &= X_L = X_{\text{rad}} + X_{\text{diff}} + X_{\text{F.K.L}} \\
m \ddot{Y}_L &= Y_L = Y_{\text{rad}} + Y_{\text{diff}} + Y_{\text{F.K.L}} \\
m \ddot{Z}_L &= Z_L = Z_{\text{restoringL}} + Z_{\text{rad}} + Z_{\text{diff}} + Z_{\text{F.K.L}} \\
I_x \ddot{P}_L - I_{xz} \ddot{R}_L &= K_L = K_{\text{restoringL}} + K_{\text{rad}} + K_{\text{diff}} + K_{\text{F.K.L}} \\
I_y \ddot{Q}_L &= M_L = M_{\text{restoringL}} + M_{\text{rad}} + M_{\text{diff}} + M_{\text{F.K.L}} \\
I_z \ddot{R}_L - I_{zx} \ddot{P}_L &= N_L = N_{\text{rad}} + N_{\text{diff}} + N_{\text{F.K.L}}.
\end{aligned} \quad (7)$$

The indices rad, diff, F.K and restoring stand for radiation, diffraction, the so-called Froude-Krylov and restoring forces and moments. Index L depicts linear approximation to the forces and moments. In the linear approximation wave excitation is assumed to comprise of the diffraction and Froude-Krylov forces and moments. The latter are evaluated from the pressures in and undisturbed oncoming wave. In the integration ship hull is assumed to have a constant

velocity  $V_s$  pointing in the  $x$ -direction and integration is conducted up to the still water level.

Mainly due to the linearity properties the solution of equations (7) is sought in the frequency domain. As a result the responses are obtained in a form of transfer functions. Thus for instance linear  $x$ -response is given by

$$x_L = x_{L0}(\omega, \mu) \cos[\omega t - k(X_G \cos \mu - Y_G \sin \mu) + \alpha_x], \quad (8)$$

where  $x_{L0}$  is motion amplitude linear in respect to wave amplitude  $a_w$ ,  $\omega$  wave frequency,  $k = \omega^2/g$  wave number and  $\alpha_x$  phase angle.

## Illustration of the two-stage approach using a single-degree-of-freedom non-linear system

When forming a mathematical model of ship, the vessel is regarded as a rigid body possessing in general six degrees of freedom. However, for the sake of simplicity the two-stage approach is explained in this section using a single-degree-of-freedom model. The model is extended to a multi-degree-of-freedom general model later on.

Let us consider a single-degree-of-freedom system given by a non-linear equation

$$m\ddot{X} + g(\dot{X}) + h(X) = F(X; t), \quad (9)$$

where  $m$  is system mass,  $t$  is time. Dots denote time derivatives. The functions  $g$  and  $h$  are in general non-linear functions of response velocity  $\dot{X}$  and displacement  $X$ . Function  $F$  is also a non-linear function of  $X$  describing external excitation to the system.

The linear version of the equation (9) is given by

$$m\ddot{x}_L + c\dot{x}_L + kx_L = F_L(t), \quad (10)$$

where  $F_L$  is a linear, independent of the response forcing function. Total response is decomposed into a linear part  $x_L$  and a non-linear portion  $x$  as follows

$$X = x_L + x. \quad (11)$$

Subtracting linear approximation (10) from the general equation (9) yields equation for the non-linear part  $x$  of the response

$$m\ddot{x} + [g(\dot{x}_L + \dot{x}) - c\dot{x}_L] + [h(x_L + x) - kx_L] = f, \quad (12)$$

where  $f = F(X;t) - F_L(t)$  is a non-linear part of the forcing function.

## Why to use two-stage approach?

The question arises why to use a two-stage approach? Why not to solve a non-linear equation of motion (6) directly in time domain by an appropriate numerical integration routine? Linear methods of the seakeeping theory are very well established. In particular hydrodynamic forces (radiation and diffraction) are well represented by the linear approximation. As a result ship motions and loads are very well established and they are given in a form of reliable transfer functions provided ship is on a straight course and motion fulfills the linearty assumptions. Direct evaluation of ship motions with an aid of a non-linear strip theory model involves certain compromises. In particular diffraction forces are often disregarded and the radiation forces are usually represented by a simplified constant added mass model. These drawbacks of the non-linear strip theory method are avoided when using a two-stage approach.

In the two-stage approach the main part of the first order, fast response in waves is given by a linear approximation. These are evaluated for an actual heading and actual position in wave. Non-linear parts of hydrostatic and hydrodynamic load, rudder and propeller forces and non-linearities of ship rigid body dynamics yield non-linear part of the first order motions and slow manoeuvring motion.

## Perturbed non-linear portion of ship motion

Having linear approximation to ship motions in waves obtained with the aid of linear seakeeping theory, non-linear part of ship motions is evaluated in the time domain. This motion takes into account non-linearities of ship hydrostatics and non-linearities of wave loads at large amplitudes of motion. The only motion component that is not decomposed into the linear and non-linear part, is surge. Total surge motion is evaluated using the 1<sup>st</sup> of equations (6). The effect of added wave resistance, propulsor action and rudder forces are included in this equation. Total ship motion, or other type of response, being a sum of linear approximation and a non-linear part is thus obtained. In other words total responses in terms of velocities are written in the following form

$$\begin{aligned} U &= ui + (v_L + v)j + (w_L + w)k \\ \Omega &= (P_L + P)i + (Q_L + Q)j + (R_L + R)k, \end{aligned} \quad (13)$$

where variables without subscripts depict non-linear part of the response. Linear approximation is evaluated with an aid of formula (8) for an actual ship position  $(X_G, Y_G)$  in wave and for an actual heading  $\mu$ .

Subtracting the equations (7) of the linear approximation model from equations (6) yields the equations for the non-linear part of response

$$\begin{aligned} m[\dot{X} + (Q_L + Q)(w_L + w) - (R_L + R)(v_L + v) + g \sin(\theta_L + \theta)] &= X \\ m[\dot{Y} + (R_L + R)u - (P_L + P)(w_L + w) - g \cos(\theta_L + \theta) \sin(\phi_L + \phi)] &= Y \\ m[\dot{Z} + (P_L + P)(v_L + v) - (Q_L + Q)u - g \cos(\theta_L + \theta) \cos(\phi_L + \phi)] &= Z \end{aligned} \quad (14)$$

$$\begin{aligned} I_x \dot{\ddot{X}} + [I_z(R_L + R) - I_{xz}(P_L + P) - I_{zy}(Q_L + Q)](Q_L + Q) - I_{xy} \dot{\ddot{Q}} \\ - I_{xz} \dot{\ddot{R}} - [I_y(Q_L + Q) - I_{yz}(R_L + R) - I_{yx}(P_L + P)](R_L + R) &= K \\ I_y \dot{\ddot{Q}} - I_{yx} \dot{\ddot{X}} + [I_x(P_L + P) - I_{xy}(Q_L + Q) - I_{xz}(R_L + R)](R_L + R) \\ - I_{yz} \dot{\ddot{R}} - [I_z(R_L + R) - I_{zx}(P_L + P) - I_{zy}(Q_L + Q)](P_L + P) &= M \\ I_z \dot{\ddot{R}} - I_{zx} \dot{\ddot{X}} + [I_y(Q_L + Q) - I_{yz}(R_L + R) - I_{yx}(P_L + P)](P_L + P) \\ - I_{zy} \dot{\ddot{Q}} - [I_x(P_L + P) - I_{xy}(Q_L + Q) - I_{xz}(R_L + R)](Q_L + Q) &= N. \end{aligned}$$

Equations (14) govern non-linear part of the rigid body motion in six degrees of freedom. In order to solve them we need to specify the non-linear part of the external (fluid) forces  $X, Y, Z$  and moments  $K, M, N$  acting on a body. These are presented in bigger detail in reference (Matusiak, 2002). Moreover we use equations (3) and (5) to express body velocities in the inertial co-ordinate system. Numerical integration of these equations together with the division of responses given by equations (13) yields the instantaneous position of ship in the inertial co-ordinate system  $X_0 Y_0 Z_0$ . Additional, thirteenth ordinary differential equation of a first order representing the action of auto-pilot is used to control the rudder angle. Integration is conducted using the 4<sup>th</sup> order Runge-Kutta scheme with an integration step being  $\Delta t = 100$  ms. Computation is conducted for a full-scale ship. Linear approximation of responses and forces is related to ship's actual position in waves. It takes into account instantaneous heading angle. The zero initial conditions are used for all equations with an exception of surge velocity, which is set initially to a prescribed ship velocity in calm water. In order to dampen the spurious transients, wave amplitude is gradually increased from zero to the prescribed final value  $a_{w,final}$  using the expression

$$\begin{aligned} a_w(t) &= a_{w,final} \left[ 1 - \left( \cos \frac{\pi t}{2T_f} \right)^2 \right] \text{ for } t < T_f \\ a_w(t) &= a_{w,final} \text{ for } t \geq T_f, \end{aligned} \quad (15)$$

where  $t$  is time and with  $T_f = 50$  seconds in full scale being used.

## Combined model of maneuvering hull and radiation forces in time domain

A quasilinear model making use of the added mass and damping concept approximates radiation forces. These forces can be expressed as

$$\begin{aligned}
 X_{\text{rad}} &= -a_{11}\dot{X} - b_{11}(u - V_s) - a_{15}\dot{Q} - b_{15}Q \\
 Y_{\text{rad}} &= -a_{22}\dot{Y} - b_{22}v - a_{24}\dot{P} - b_{24}P - a_{26}\dot{R} - b_{26}R \\
 Z_{\text{rad}} &= -a_{33}\dot{W} - b_{33}w - a_{35}\dot{Q} - b_{35}Q \\
 K_{\text{rad}} &= -a_{44}\dot{P} - b_{44}P - a_{46}\dot{R} - b_{46}R - a_{42}\dot{Y} - b_{42}v \\
 M_{\text{rad}} &= -a_{55}\dot{Q} - b_{55}Q - a_{53}\dot{W} - b_{53}w - a_{51}\dot{X} - b_{51}(u - V_s) \\
 N_{\text{rad}} &= -a_{66}\dot{R} - b_{66}R - a_{64}\dot{P} - b_{64}P - a_{62}\dot{Y} - b_{62}v.
 \end{aligned} \tag{16}$$

In equations (16)  $a_{ij}$  and  $b_{ij}$  depict added masses and damping coefficients referred to the origin located in the center of gravity (G in Fig. 1). These are frequency dependent values. In the present method these coefficients are evaluated by a standard linear seakeeping theory based computer program (Journée, 1992). Note that radiation forces are oriented in the body-fixed coordinate system.

### Memory effect included using the retardation function concept

Time domain approach requires the so-called convolution integral representation of the radiation forces (Cummins, 1962). In this time approach radiation forces vector  $\mathbf{X}_{\text{rad}}$  is represented by an expression:

$$\mathbf{X}_{\text{rad}}(t) = -\mathbf{a}_{\infty}\ddot{\mathbf{x}}(t) - \int_{-\infty}^t \mathbf{k}(t - \tau)\dot{\mathbf{x}}(\tau) d\tau, \tag{17}$$

where  $\mathbf{a}_{\infty}$  is the matrix comprising of the added masses coefficients for an infinite frequency and  $\mathbf{x}$  is the response vector. Matrix function  $\mathbf{k}$  is the so-called retardation function which takes into account the memory effect of the radiation forces. This function can be evaluated as follows

$$\mathbf{k}(t) = \frac{2}{\pi} \int_0^{\infty} \mathbf{b}(\omega) \cos(\omega t) d\omega, \tag{18}$$

where  $\mathbf{b}$  is the frequency dependent added damping matrix. In order to take into account the manoeuvring hull forces four components of the  $\mathbf{b}$  matrix are



modified as follows. Terms of sway, yaw and their coupling terms of the linear hull forces model are subtracted from the corresponding elements of matrix  $\mathbf{b}$ , ie

$$\begin{aligned} b'_{22}(\omega) &= b_{22}(\omega) - Y_V \\ b'_{66}(\omega) &= b_{66}(\omega) - \square_R \\ b'_{62}(\omega) &= b_{62}(\omega) - \square_V \\ b'_{26}(\omega) &= b_{26}(\omega) - Y_{R.}. \end{aligned} \quad (19)$$

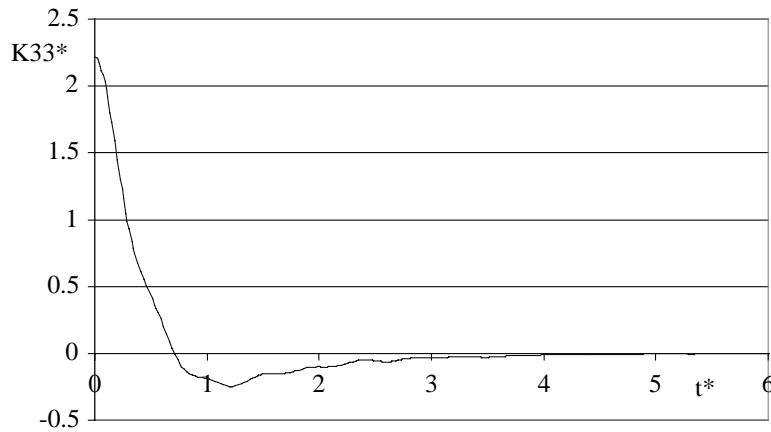
The  $\mathbf{k}(t)$  functions have to be evaluated only once before the simulation. The Fast Fourier Transform algorithm is used when evaluating discrete values of the retardation functions as follows (Matusiak, 2001)

$$K_{k,ij}(k\Delta t) = \frac{N\Delta\omega}{\pi} \mathbf{FFT}(g_{ij}(x)), \quad (20)$$

where the original added damping discrete functions are substituted by a 'double-sided function'  $g(x)$  as follows:

$$\begin{aligned} g_{ij}(x) &= b_{ij}(x) \text{ for } x = \Delta\omega, \dots, \Delta\omega N/2 \\ g_{ij}(N\Delta\omega - x) &= b_{ij}(x) \text{ for } x = 0, \dots, \Delta\omega(N/2 + 1). \end{aligned} \quad (21)$$

Note that as a result the retardation function 16 is obtained at  $N/2$  discrete time instants with a time step  $\Delta t$ . FFT analysis is conducted with  $N = 2048$ . As a result the retardation functions are represented by 1024 discrete values covering the period of 102.4 seconds. An example of the retardation function for heave is given below.

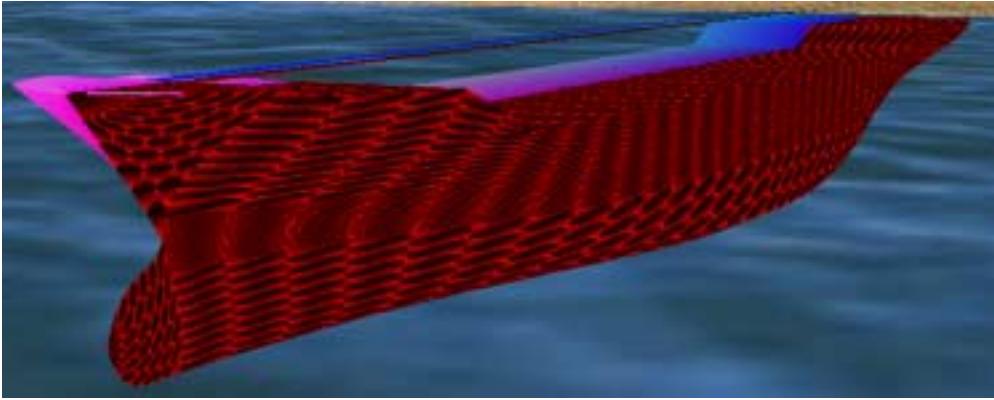


**Fig. 2** *Non-dimensional heave memory (retardation) function*

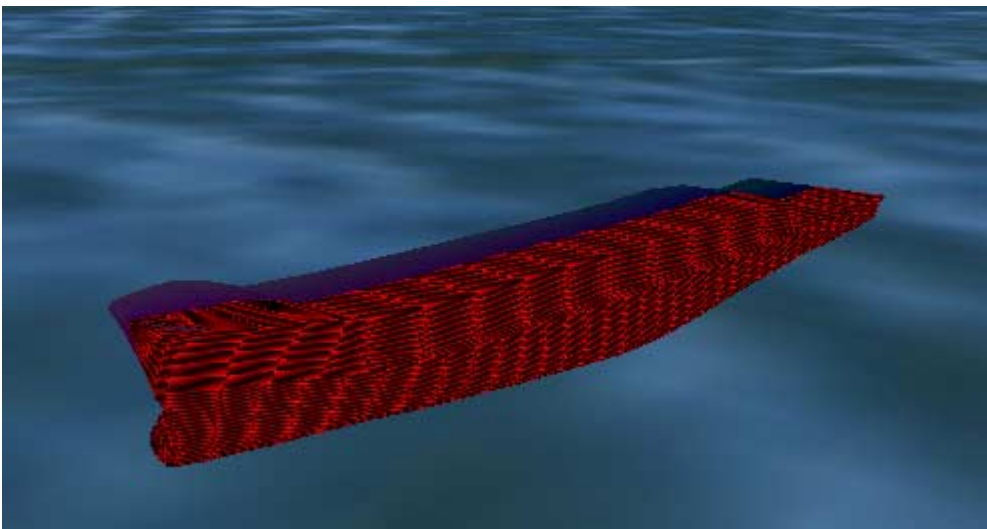
$$K_{33}^* = K_{33} / (m \sqrt{g/L}) \text{ as a function of non-dimensional time}$$
$$t^* = t / \sqrt{g/L}, \text{ where } L \text{ is waterline length of ship.}$$

## Examples of ship motion simulation and method validation

Two ship cases were investigated within the benchmark study of ITTC. First one, ship A1 (see Fig. 4), is a containership of waterline length of 150 m and low metacentric height ( $GM_0 = 0.15$  m). Second vessel (see Fig. 5) is a model of seiner of the length  $L_{pp} = 35.68$  [m]. Both models were run in regular waves, different headings and Froude numbers.



**Fig. 4** *Three dimensional panel model of ship A1.*



**Fig. 5** *Three dimensional panel model of ship A2.*

The summary of model test results and simulation results are given in Tables 1 and 2.

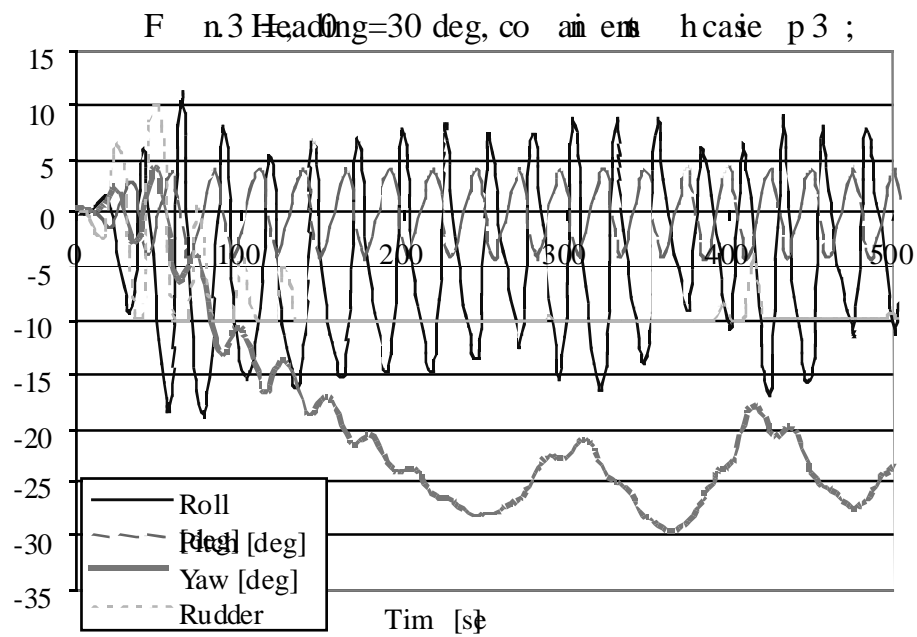
**Table 1** Summary of the results for containership (Ship A1).

Case	$\lambda/L_{pp}$	$2a_w/\lambda$	<b>Fn</b>	<b>Heading</b> [deg]	<b>Experiment</b>	<b>Computed</b>
1	1.5	1/25	0.2	0	Parametric roll resonance, <b>capsize</b>	Parametric roll resonance, non-capsizing
2	1.5	1/25	0.2	45	no- capsizing	no- capsizing
3	1.5	1/25	0.3	30	no- capsizing	no- capsizing
4	1.5	1/25	0.4	30	<b>capsize</b>	<b>capsize</b>

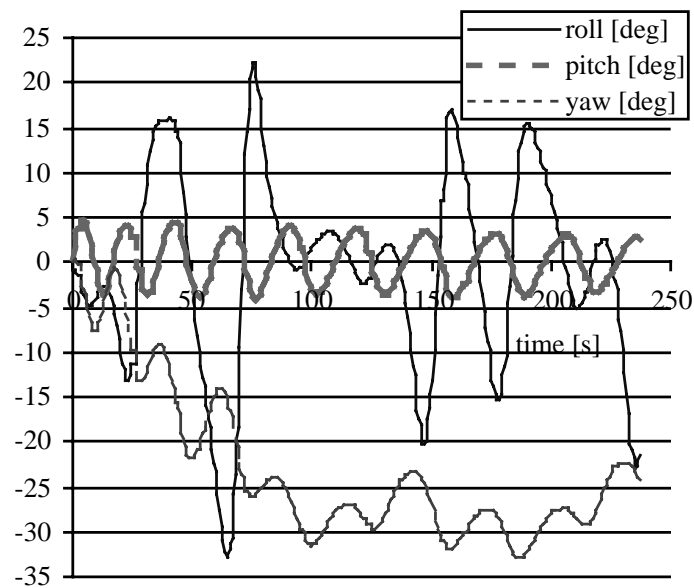
**Table 2** Summary of the results for Seiner (Ship A2).

Case	$\lambda/L_{pp}$	$2a_w/\lambda$	<b>Fn</b>	<b>Heading</b> [deg]	<b>Experiment</b>	<b>Computed</b>
A	1.637	0.1	0.3	-30	no-capsizing	no-capsizing
B	1.637	0.1	0.43	-10	surfing, <b>capsize</b>	surfing
C	1.127	0.115	0.3	-30	no- capsizing	no- capsizing
D	1.127	0.115	0.43	-30	<b>capsize</b>	<b>capsize</b>

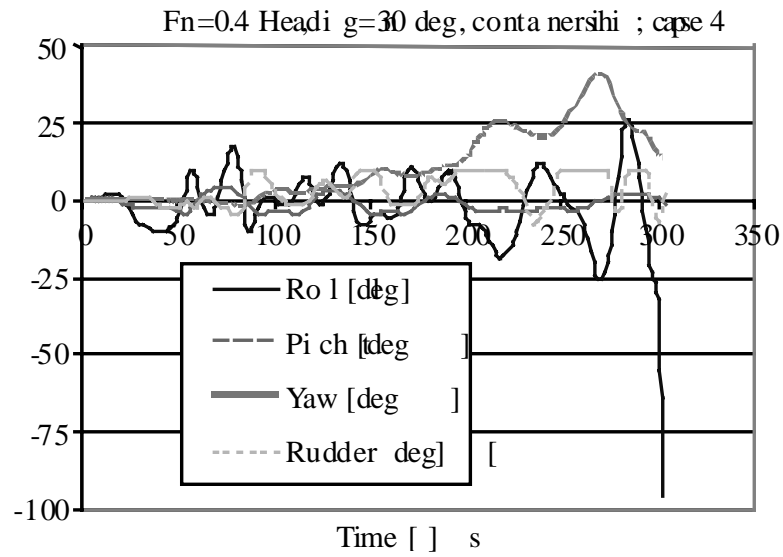
Selected time-histories of the angular motions of both vessels are presented in Figs. 6-13.



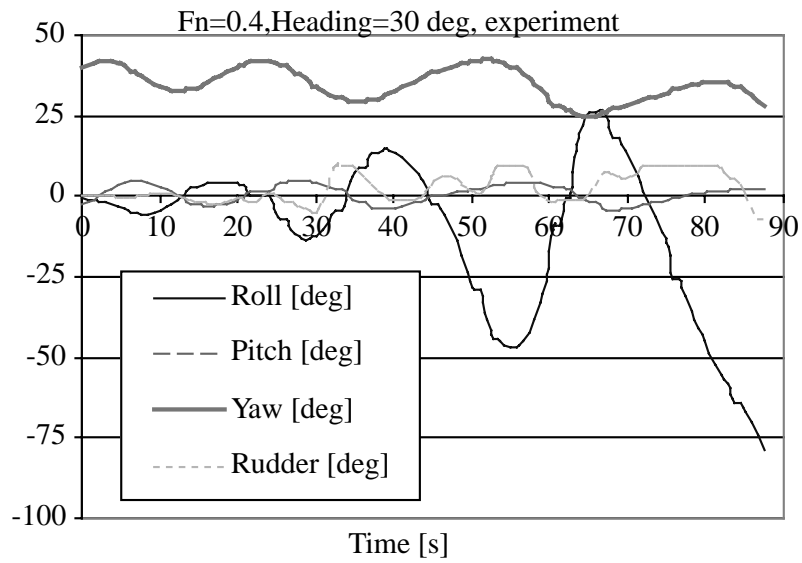
**Fig. 6.** Simulated angular motions of ship in regular quartering waves; Case 3 (heading = 30 [deg]). Ship speed is  $F_n = 0.3$ .



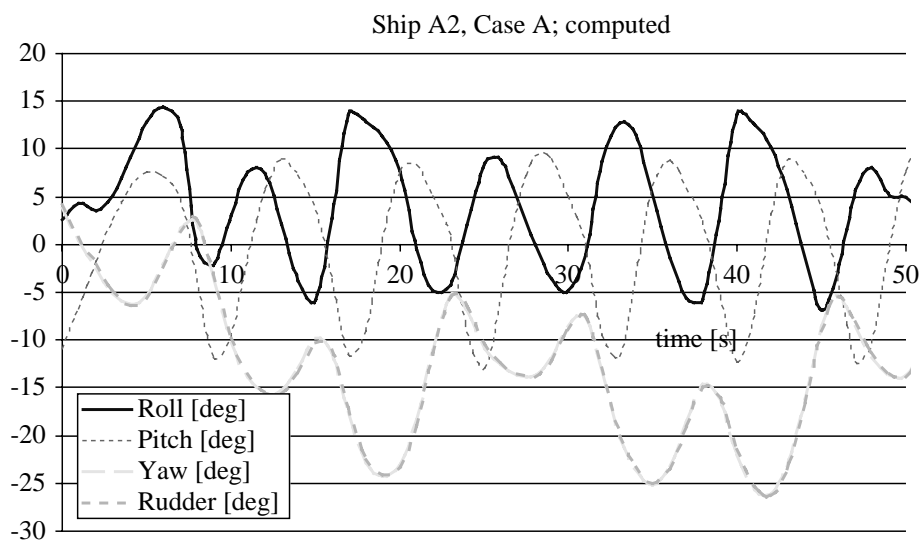
**Fig. 7** Angular motions of ship in regular quartering waves of containership; Case 3, (heading = 30 [deg]). Ship speed is  $F_n = 0.3$ . Model test result scaled to full-scale and yaw defined as a deviation from the initial course.



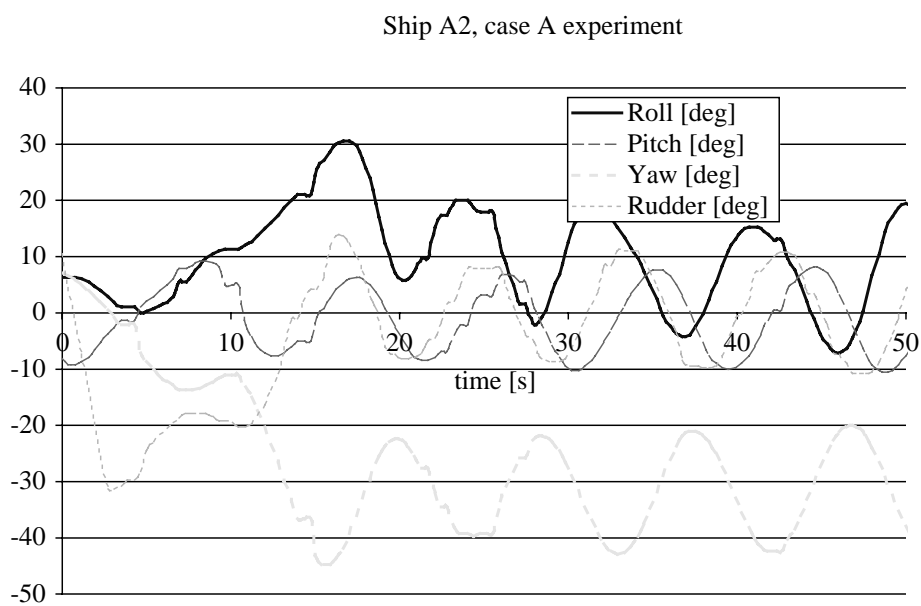
**Fig. 8** Containership running at  $F_n = 0.4$  capsizes in regular quartering regular waves (heading 30 [deg]; case 4). Result of simulation.



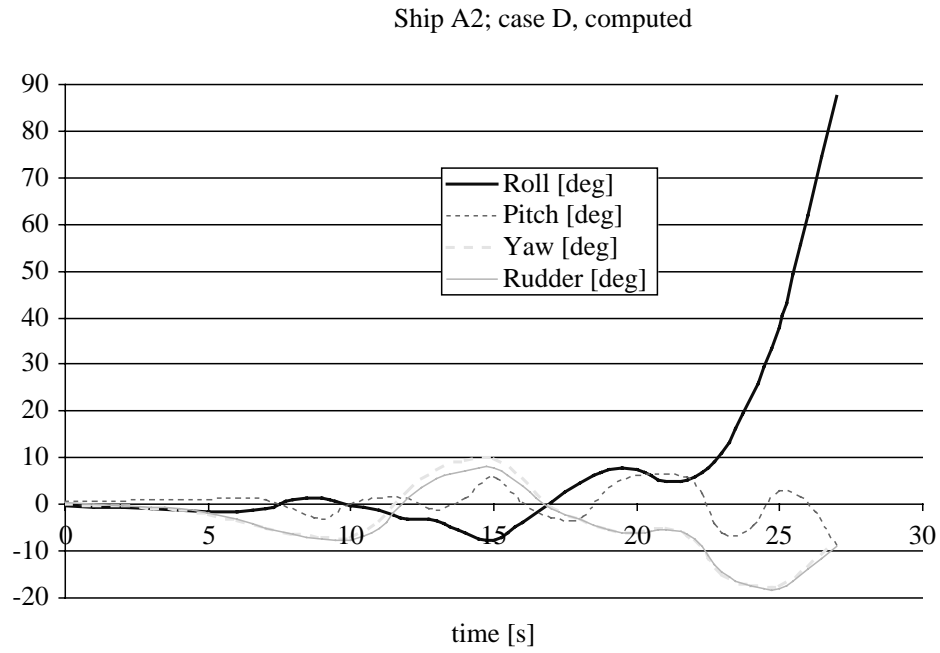
**Fig. 9** Model of containership running at  $F_n = 0.4$  capsizes in regular quartering regular waves (heading 30 [deg]). Model test result scaled to full-scale. [5].



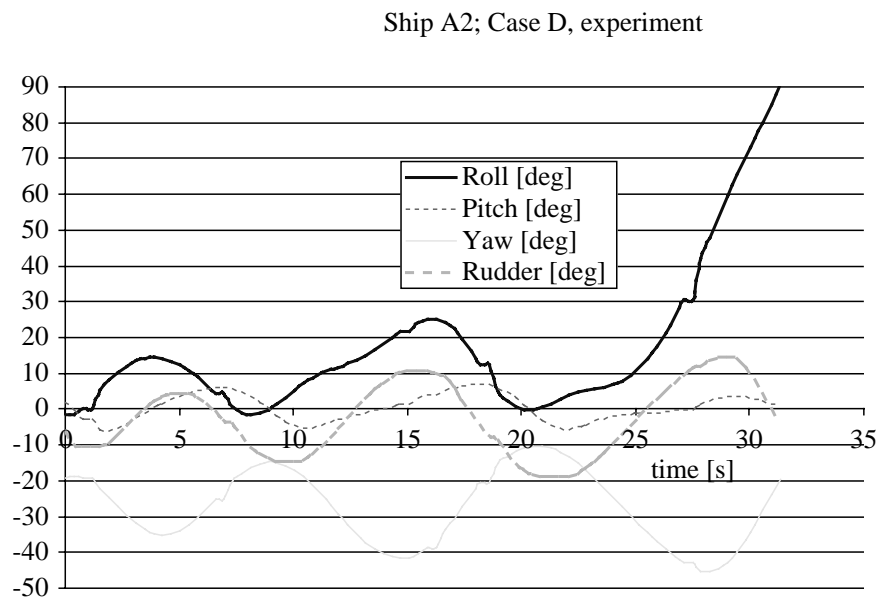
**Fig. 10** Angular motions of Seiner in regular quartering waves of; Case A, (heading =  $-30$  [deg]). Ship speed is  $F_n = 0.3$ . Simulated result.



**Fig. 11** Angular motions of Seiner in regular quartering waves; Case A, (heading =  $-30$  [deg]). Ship speed is  $F_n = 0.3$ . Measured result.



**Fig. 12** Ship A2 (Seiner) capsizes in regular quartering waves; Case D, (heading =  $-30$  [deg]). Ship speed is  $Fn = 0.43$ . Simulated result.



**Fig. 13** Ship A2 (Seiner) capsizes in regular quartering waves; Case D, (heading =  $-30$  [deg]). Ship speed is  $Fn = 0.43$ . Measured in model scale.



## Conclusions

The combined model of manoeuvring and non-linear seakeeping yields ship motions which at least qualitatively agree with the model test experiments.

There are several possible reasons for a certain disagreement of simulation and model test results.

Initial conditions are set to zero in the simulations. Measurements of ship motions in model tests are started at a certain instant with the initial conditions which are not very well known. Usually both waves and ship motions are already well developed at the beginning of an experiment. As the initial conditions have a big influence on the response on a non-linear system, this may affect the comparison of results.

At high Froude numbers ( $F_n > 0.4$ ) an effect of dynamic lift may be important on ship static stability. For the time being this effect is not taken into account in the presented method.

Manoeuvring hull forces are regarded as linear ones and as independent of the ship first order motions in waves. This assumption may be not good when considering a loss of dynamic stability in waves where a change of ship's course may be very rapid.

The above mentioned simplifications of the presented method have to further studied. Nevertheless the method, already at the present stage of development, may be used as a tool to evaluate ship dynamic stability and to simulate ship manoeuvring in waves.

The method can be easily further developed to consider other matters relevant to ship design. A possibility to evaluate non-linear sectional loads in terms of total shear forces, bending and torsional moments is being developed. The aim of this study is to reveal unsymmetry of sagging and hogging of total sectional loads in waves. Modelling the action of the turnable Azipod-type z-drive unit on manoeuvring is considered, too.

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